

Hvordan referanser kan etablere  
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(og en masteroppgave)

How references may establish a  
sound foundation of an article  
(and maybe a thesis)

# On the Minimality of Finite Automata and Stream X-machines for Finite Languages

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A cover automaton of a finite language  $L$  is a finite automaton that accepts all words in  $L$  and possibly other words that are longer than any word in  $L$ . An algorithm for constructing a minimal cover automaton of a finite language  $L$  is given in a recent paper. This paper goes a step further by proposing a procedure for constructing all minimal cover automata of a given finite language  $L$ . The concept of cover automaton is then generalized to a form of extended finite automaton, the stream X-machine, and the procedure is extended to this more general model.

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## 1. INTRODUCTION

Finite automata [1, 2, 3] are widely used in computer systems of computing, ranging from lexical analysis to protocol testing. Finite automata are known to accept all regular languages [4, 5]. However, in many applications, only finite automata only finite languages are used. The number of states of a finite automaton (FA) that accepts a finite language is at least one more than the length of the longest word in the language and may be exponentially large. On the other hand, if we do not restrict the number of states to accept only the given finite language but allow extra words that are longer than the longest word in the language, then the number of its states may be reduced. In most applications the maximum length of words in the language is known and the system of the length of the words processed, so the number of states will usually be adequate. This is the reason why finite automata for finite languages are used.

Informally, a cover automaton of a finite language  $L$  is an FA that accepts all words in  $L$  and possibly other words that are longer than any word in  $L$ . A minimal cover automaton of  $L$  is a cover automaton of  $L$  having the least number of states. In many cases, a minimal cover automaton of  $L$  has a much smaller size than the minimal automaton that accepts  $L$ .

The concept of minimal cover automaton of a finite language is introduced in [6] and it is shown that there may be several minimal cover automata of the same language that are not isomorphic. Furthermore, [6] provides an algorithm that, for a finite language  $L$  (given as an FA that accepts  $L$  or as a cover automaton of  $L$ ), constructs a minimal cover automaton of the language. An improved algorithm (in terms of complexity) is also presented in [7].

This paper goes a step further by giving a procedure for constructing all minimal cover automata of a given finite language.

# Finite automata [1, 2, 3]

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Investigating and techniques for refining given specifications into more complex, more detailed implementation-oriented versions have been developed [14, 15]. Furthermore, several models of communicating SXMs have been devised and used in real applications [16, 17, 18].

One of the strengths of using SXMs to specify a system is that it is possible to derive test sets from an SXM specification which, if satisfied, guarantee, under certain constraints, the correctness of the implementation with respect to the specification [10, 19, 20, 21]. Among these constraints are the so-called 'design for test conditions' that the SXM specification has to meet: input-completeness and output-distinguishability [10, 19]. The class of SXMs that meet these conditions is therefore of particular interest and has

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A specification method, especially for interactive systems. A tool to support the creation of SXM specifications has been constructed [13]. The refinement of SXMs has been investigated and techniques for refining given specifications into more complex, more detailed implementation-oriented versions have been developed [14, 15]. Furthermore, several models of communicating SXMs have been devised and used in real applications [16, 17, 18].

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